Přenosová funkce

I. Vnější popis

$$y''(t) + 2a\,y'(t) + (a^2+b^2)\,y(t) = u(t) \qquad y(0) = c_1 \qquad y'(0) = c_2$$

$$p^2 Y(p) - p\cdot y(0) - y'(0) + 2a\left[p\cdot Y(p) - y(0)\right] + (a^2+b^2)\,Y(p) = U(p)$$

$$p^2 Y(p) - c_1 p - c_2 + 2ap\,Y(p) - 2a c_1 + (a^2+b^2)\,Y(p) = U(p)$$

$$Y(p)\left[p^2 + 2ap + (a^2+b^2)\right] = U(p) + c_1(p+2a) + c_2$$

$$Y(p) = \frac{U(p) + c_1(p+2a) + c_2}{p^2 + 2ap + (a^2+b^2)}$$

$$Y(p) = \overset{a)}{\frac{U(p)}{p^2+2ap+(a^2+b^2)}} + \overset{b)}{\frac{c_1(p+2a)+c_2}{p^2+2ap+(a^2+b^2)}}$$

$$H(p) = \frac{Y(p)}{U(p)} = \frac{\frac{\cancel{U(p)}}{p^2+2ap+(a^2+b^2)}}{\cancel{U(p)}} + \frac{\frac{c_1(p+2a)+c_2}{p^2+2ap+(a^2+b^2)}}{U(p)}$$

$$u \longrightarrow \boxed{\mathcal{T}(*)} \longrightarrow y$$

$$y(t) = \int_0^\infty u(\tau)\,h(t-\tau)\,d\tau = \int_0^\infty u(t-\tau)\,h(\tau)\,d\tau$$

Laplaceova tr.

$$Y(p) = H(p)\cdot U(p)$$

$$H(p) = \frac{Y(p)}{U(p)}$$

přenosová funkce

a) $\dfrac{U(p)}{p^2+2ap+(a^2+b^2)}$ USTÁLENÁ SLOŽKA

b) $\dfrac{c_1(p+2a)+c_2}{p^2+2ap+(a^2+b^2)}$ PŘECHODOVÁ SLOŽKA

Přenosovou funkci můžeme vyjádřit JOUZE pro ustálenou složku

$\Rightarrow$ počáteční podmínky $= 0$

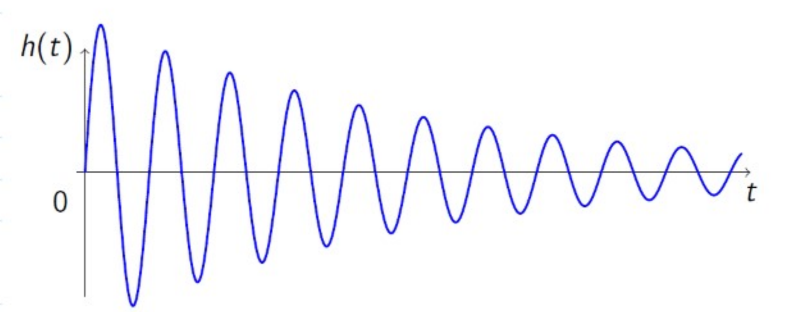
$$c_1 = 0\,;\ c_2 = 0$$

$$H(p) = \frac{1}{p^2+2ap+(a^2+b^2)}$$

$\mathcal{L}^{-1}\{H(p)\} = h(t) \rightarrow$ impulsní odezva

$$h(t) = \mathcal{L}^{-1}\left\{\frac{1}{p^2+2ap+(a^2+b^2)}\right\} = \frac{1}{b}\,e^{-at}\sin bt$$

$$\frac{1}{(p+a)^2+b^2} \quad \Big\{ \quad e^{-\alpha t}\sin\omega t \quad \Big| \quad \frac{\omega}{(p+\alpha)^2+\omega^2} \Big\} = \frac{1}{b}\,\frac{b}{(p+a)^2+b^2}$$



Přechodová odezva (odezva na $\mathbb{1}(t)$)

$$S(p) = \frac{1}{p} \cdot H(p) \qquad s(t) = \mathcal{L}^{-1}\{S(p)\}$$

$$Y(p) = H(p)\,U(p)$$

- impulsní odezva (odezva na $\delta(t)$)

$$h(t) = \mathcal{L}^{-1}\{H(p) \cdot \underbrace{\mathcal{L}\{\delta(t)\}}_{1}\}$$

- přechodová od. na $\mathbb{1}(t)$

$$s(t) = \mathcal{L}^{-1}\{H(p) \cdot \underbrace{\mathcal{L}\{\mathbb{1}(t)\}}_{\frac{1}{p}}\}$$

II. VNITŘNÍ POPIS

$$H(p) = \frac{Y(p)}{U(p)}$$

$$A = [n \times n] \qquad p = [1 \times 1] \qquad p \cdot I = p \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

$$x'(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$

$$p\,X(p) - x(0) = A\,X(p) + B\,U(p)$$
$$Y(p) = C\,X(p) + D\,U(p)$$

$$p\,X(p) - A\,X(p) = x(0) + B\,U(p)$$
$$(pI - A)\,X(p) = x(0) + B \cdot U(p)$$
$$X(p) = (pI - A)^{-1} x(0) + (pI - A)^{-1} B \cdot U(p)$$

$(pI - A)^{-1}$ → inverzní matice

$$Y(p) = C\left[(pI - A)^{-1} x(0) + (pI - A)^{-1} B \cdot U(p)\right] + D\,U(p) =$$
$$= \underbrace{C(pI - A)^{-1} x(0)}_{\text{přechodová složka}} + \underbrace{C(pI - A)^{-1} B\,U(p)}_{\text{ustálená složka}} + D\,U(p)$$

p.p. nulové !

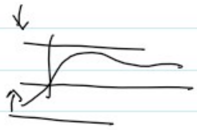
$$Y(p) = C(pI - A)^{-1} B\,U(p) + D\,U(p) = \left[C(pI - A)^{-1} B + D\right] U(p)$$

$$H(p) = \frac{Y(p)}{U(p)} \qquad \boxed{H(p) = \mathbf{C}\,(p\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B} + \mathbf{D}}$$

$$(pI - A)^{-1} = \frac{adj(pI - A)}{det(pI - A)}$$

STABILITA

BIBO (bounded input , bounded output)



$$H(p) \xrightarrow{\mathcal{L}^{-1}} h(t)$$

$$\lim_{t \to \infty} h(t) \begin{cases} 0 \ ; & \text{stabilní systém} \\ a \ ; & \text{mez stability} \\ \pm\infty \ ; & \text{nestabilní systém} \end{cases}$$

př.

$$y''(t) + 4y'(t) + 3y(t) = \underbrace{\sin(t)}_{u(t)} \qquad \text{p.p.} \quad y(0) = 1 \ ; \ y'(0) = -1$$

$$y''(t) + 4y'(t) + 3y(t) = \frac{\sin(t)}{u(t)} \qquad \text{p.p.} \quad y(0) = 1 \,;\; y'(0) = -1$$

přenosová funkce — nulové p.p. $y(0) = 0 \,;\; y'(0) = 0$

$$p^2 Y(p) + 4pY(p) + 3Y(p) = U(p) \qquad \Big\{ \; H(p) = \frac{Y(p)}{U(p)}$$

$$H(p) = \frac{1}{p^2 + 4p + 3}$$

impulsní odezva

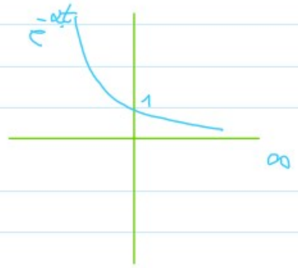
$$\mathcal{L}^{-1}\{H(p)\} \qquad \frac{1}{p^2+4p+3} = \frac{1}{(p+1)(p+3)} = \frac{A}{p+1} + \frac{B}{p+3}$$

$$A = \frac{1}{p+3}\Big|_{p=-1} = \frac{1}{2} \qquad B = \frac{1}{p+1}\Big|_{p=-3} = -\frac{1}{2}$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{2}\frac{1}{p+1} - \frac{1}{2}\frac{1}{p+3} \right\} = \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}$$

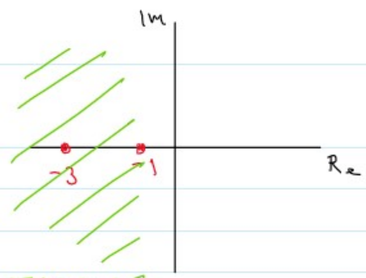
Stabilita

$$\lim_{t\to\infty} \underbrace{\frac{1}{2}e^{-t}}_{\to 0} - \underbrace{\frac{1}{2}e^{-3t}}_{\to 0} = 0$$



SYSTÉM JE STABILNÍ

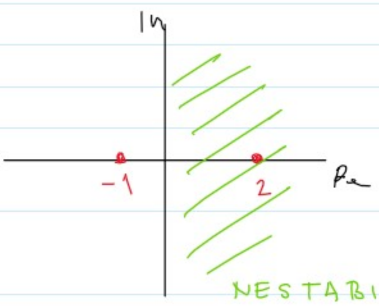
p-rovina



STABILNÍ

$$H(p) = \frac{1}{p^2+4p+3} = \frac{1}{(p+1)(p+3)}$$

$p_1 = -1$ $\qquad$ $p_2 = -3$



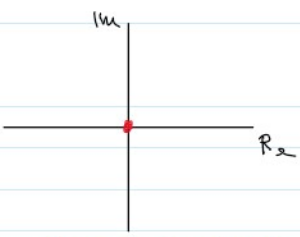
NESTABILNÍ

$$H(t) = \frac{1}{p^2 - p - 2} = \frac{1}{(p+1)(p-2)} = \dots$$

$p_1 = -1$ $\qquad$ $p_2 = 2$

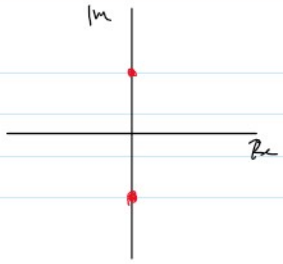
$$\mathcal{L}^{-1}\{H(p)\} = \mathcal{L}^{-1}\left(A\frac{1}{p+1} + B\frac{1}{p-2}\right) =$$

$$= \underbrace{A \cdot e^{-t}}_{\substack{0 \\ t\to\infty}} + \underbrace{B \cdot e^{2t}}_{\substack{\infty \\ t\to\infty}}$$

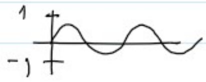


MEZ STABILITY

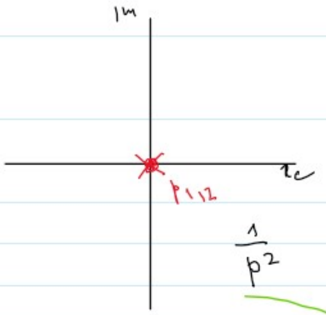
$$H(p) = \frac{1}{p} \qquad \mathcal{L}^{-1}\{H(p)\} = \mathbb{1}(t)$$



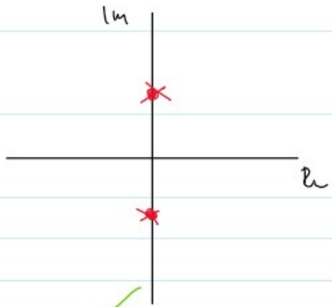
$$H(p) = \frac{1}{p^2+1} \quad \Big\{ \quad \sin\omega t \quad \Big| \quad \frac{\omega}{p^2+\omega^2}$$



MEZ STABILITY



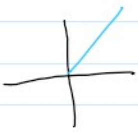
$\frac{1}{p^2}$



$\sim$ $t \sin$
$t \cos$

$t^n \quad \Big| \quad \frac{n!}{p^{n+1}}$

NESTABILNÍ



STABILITA VNITŘNÍ POPIS

př.

$$y''(t) + 4y'(t) + 3y(t) = \underbrace{\sin(t)}_{u(t)} \qquad \text{p.p.} \quad y(0) = 1;\ y'(0) = -1$$

$$x_1(t) = y(t) \qquad x_1'(t) = y'(t) = x_2(t)$$
$$x_2(t) = y'(t) \qquad x_2'(t) = y''(t) = -3x_1(t) - 4x_2(t) + \sin(t)$$

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sin(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$H(p) = \mathbf{C}\,(p\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B} = \mathbf{C}\,\frac{\operatorname{adj}(p\mathbf{I} - \mathbf{A})}{\det(p\mathbf{I} - \mathbf{A})}\,\mathbf{B}$$

jmenovatel přenosové funkce

$\det(pI - A)$

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$$

$$pI = p \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}$$

$$(pI - A) = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} p & -1 \\ 3 & p+4 \end{bmatrix}$$

$$\det \begin{pmatrix} p & -1 \\ 3 & p+4 \end{pmatrix} = p(p+4) - (-1)(3) = p^2 + 4p + 3$$

$$(pI - A) = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} p & -1 \\ 3 & p+4 \end{bmatrix}$$

$$\det \begin{pmatrix} p & -1 \\ 3 & p+4 \end{pmatrix} = p(p+4) - (-1)(3) = p^2 + 4p + 3$$

Konzultace:

$$y^{(5)}(t) + 2y^{(3)}(t) + y''(t) - 6 \cdot y'(t) + y(t) = u(t)$$

$$H(p) = \frac{1}{p^5 + 2p^3 + p^2 - 6p + 1} + \frac{\text{poč. podmínky}}{p^5 + 2p^3 + p^2 - 6p + 1}$$